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Hadronic cross sections in electron–positron annihilation with tagged photon

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Abstract: Events with tagged photons in the process of electron–positron annihilation into hadrons are considered. The initial state radiation is suggested to scan the hadronic cross section with the energy. QED radiative corrections are taken into account. The results for the total and exclusive cross sections are given in an analytic form. Some numerical estimates are presented.

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Contents

1. Introduction

Experiments with tagged photons, radiated from the initial state in electron–proton and electron–positron collisions, can become particularly attractive. The reason is that these radiative processes will permit to extract information about the final states at continuously varying values of the collision energy. To investigate deep inelastic scattering the authors of ref. [1] suggested to use radiative events instead of running colliders at reduced beam energies. The method takes advantage of a photon detector (PD) placed in the very forward direction, as seen from the incoming electron beam. The effective beam energy, for each radiative event, is determined by the energy of the hard photon is observed in PD. In fact, radiative events were already used to measure the structure function F_2 down to $Q^2 \geq 1.5$ GeV^2 [2]. The specific theoretical work concerns the evaluation of QED radiative corrections [3, 4] to the radiative Born cross section. With an accurate determination of the cross sections and of the possible sources of background we believe that the use of radiative events may become particularly useful to carry investigations at various present and future machines.

The important role of the initial state radiation in the process of electron–positron annihilation was underlined in a series of papers by V.N. Baier and V.A. Khoze [5], where the radiative process was studied in detail in the Born approximation. In these papers the mechanism of returning to a resonant region was discovered.

This mechanism consists in the preferable emission of photons from the initial particles, which provides a resonant kinematics of a subprocess. A utilization of radiative events can become a common type of investigations at various machines.

In this paper we derive explicit formulae for the spectrum of tagged photons. The calculations are performed having an accuracy of the per–mille order as an aim. Formulae can be used at electron–positron colliders to investigate, for instance, hadronic final states at intermediate energies. A measurement of the total hadronic cross section at low energies is essential for an high precision test of the Standard Model particularly for a precise determination of the fine structure constant $\alpha_{QED}(M_Z)$ and of the muon anomalous magnetic moment $(g-2)_\mu$. The largest contribution to the errors for these quantities comes from the large indetermination still present on the measurement of the total hadronic cross section in electron–positron annihilation at center–of–mass energies of a few GeVs. We will consider here the radiatively corrected cross section for the electron–positron annihilation process

$$e^-(p_1) \ + \ e^+(p_2) \ \longrightarrow \ \gamma(k) \ + \ H(q)\,, \qquad k = (1-z)p_1\,, \tag{1.1}$$

where H is a generic hadronic state. The hard photon hitting the photon detector has a momentum k and an energy fraction $1-z$ with respect to the beam energy. In the following we assume that the photon detector is placed along the electron beam direction, and has an opening angle $2\theta_0 \ll 1$, such that $\varepsilon^2\theta_0^2 \gg m^2$, with m the electron mass, and ε the beam energy. To evaluate the process with an accuracy of the per mille requires a careful investigation of the radiative corrections. This paper is organized as follows. In section 2 we consider the cross section of the process (1.1) in the Born approximation. We give formulae suitable to study as differential distributions in hadronic channels, as well as the total (in terms of quantity R) and inclusive (in terms of hadron fragmentation functions) hadronic cross sections. In section 3 we calculate separate contributions into radiatively corrected cross section of process (1.1) within the next–to–leading accuracy. In section 3.1 the contribution due to virtual and soft photon emission is investigated. In section 3.2 the case, when additional hard photon hits a photon detector is considered. In section 3.3 the contribution due to hard photon emission, which does not hit a photon detector is derived. In section 4 we sum up all the contributions and give the final result. In Conclusions we summarize the results and give some numerical illustrations.

2. The Born approximation

In order to obtain the Born approximation for the cross section of the process (1.1), when the PD is placed in front of electron (positron) beam, we can use the quasireal electron method [6]. It gives

$$\mathrm{d}\sigma(k, p_1, p_2) = \mathrm{d}W_{p_1}(k)\sigma_0(p_1 - k, p_2)\,, \tag{2.1}$$

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where $\mathrm{d}W_{p_1}(k)$ is the probability to radiate photon with energy fraction $1-z$ inside a narrow cone with the polar angle not exceeding $\theta_0 \ll 1$ around the incoming electron, and $\mathrm{d}\sigma_0$ is the differential cross section for the radiationless process of electron–positron annihilation into hadrons at the reduced electron beam energy. The form of both, $\mathrm{d}W_{p_1}(k)$ and $\sigma_0(p_1-k, p_2)$ is well known:

$$\mathrm{d}W_{p_1}(k) = \frac{\alpha}{2\pi} P_1(z, L_0)\mathrm{d}z\,, \quad P_1(z, L_0) = \frac{1+z^2}{1-z} L_0 - \frac{2z}{1-z}\,, \quad L_0 = \ln\frac{\varepsilon^2\theta_0^2}{m^2}\,. \tag{2.2}$$

We need further the general form of the lowest order cross section σ_0 for the process $e^+(z_1 p_2) + e^-(zp_1) \to$ hadrons boosted along the beam axis ($\boldsymbol{p}_1$):

$$\sigma_0(z, z_1) = \frac{8\pi^2\alpha^2}{q^2|1-\Pi(q^2)|^2}\int T(q)\mathrm{d}\Gamma(q)\,, \tag{2.3}$$

where

$$\begin{gathered}
\mathrm{d}\Gamma(q) = (2\pi)^4\delta(q - \textstyle\sum q_j)\prod \frac{\mathrm{d}^3 q_j}{2\varepsilon_j(2\pi)^3}\,, \\
\begin{aligned}
T(q) &= \frac{L_{\rho\sigma}H_{\rho\sigma}}{(q^2)^2}\,, & L_{\rho\sigma} &= \frac{q^2}{2}\widetilde{g}_{\rho\sigma} + 2z^2\widetilde{p}_{1\rho}\widetilde{p}_{1\sigma}\,, \\
q &= zp_1 + z_1 p_2\,, & q^2 &= s z_1 z\,, \\
\widetilde{g}_{\rho\sigma} &= g_{\rho\sigma} - \frac{q_\rho q_\sigma}{q^2}\,, & \widetilde{p}_{1\rho} &= p_{1\rho} - \frac{p_1 q}{q^2}q_\rho\,,
\end{aligned}
\end{gathered} \tag{2.4}$$

and q is the full 4–momentum of final hadrons, q_j is 4–momentum of an individual hadron, $s = 2p_1p_2 = 4\varepsilon^2$ is the full center–of–mass energy squared, and $H_{\rho\sigma}$ is the hadronic tensor. The vacuum polarization operator $\Pi(q^2)$ of the virtual photon with momentum q is a known function [7] and will not be specified here.

The tensors $H_{\rho\sigma}$ and $L_{\rho\sigma}$ obey the current conservation conditions once saturated with the 4–vector q. The differential cross section with respect to the tagged photon energy fraction z can be obtained by performing the integration on the hadrons phase space. It takes the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{\alpha}{2\pi}P_1(z, L_0)\;\sigma_0(z, 1)\,. \tag{2.5}$$

Each hadronic state is described by its own hadronic tensor. The cross section in eqs. (3) and (4) is suitable for different uses and, as mentioned above, it can be used to check different theoretical predictions.

The sum of the contributions of all hadronic channels, by means of the relation

$$\sum_h \int H_{\rho\sigma}\mathrm{d}\Gamma = f_h(q^2)\widetilde{g}_{\rho\sigma}\,, \tag{2.6}$$

can be expressed in terms of the ratio of the total cross section for annihilation into hadrons and muons $R = \sigma_h/\sigma_\mu$. For the $\mu^+\mu^-$ final state we get

$$f_\mu = \frac{q^2}{6\pi}K(q^2)\,, \qquad K(q^2) = \left(1 + \frac{2m_\mu^2}{q^2}\right)\sqrt{1 - \frac{4m_\mu^2}{q^2}}\,,$$

and so,

$$f_h(q^2) = \frac{q^2 R(q^2)}{6\pi} K(q^2) \,. \tag{2.7}$$

Substituting this expression into the right hand side of eqs. (3,4) results in the replacement $\sigma_0(z,z_1) = R(q^2)4\pi\alpha^2 K(q^2)/(3q^2)$.

In experiments of semi–inclusive type one fixes an hadron with 3-momentum $\boldsymbol{q}_1$ energy ϵ_1 and mass M in every event and sum over all the rest. In this case instead of eq. (2.6) we will have (similarly to the Deep Inelastic Scattering (DIS) case [4]):

$$\begin{aligned}
&\sum_{h'}\int H_{\rho\sigma}\mathrm{d}\Gamma = H^{(1)}_{\rho\sigma}\frac{\mathrm{d}^3 q_1}{2\varepsilon_1(2\pi)^3}\,, && (2.8)\\
&H^{(1)}_{\rho\sigma} = F_1(\eta,q^2)\widetilde{g}_{\rho\sigma} - \frac{4}{q^2}F_2(\eta,q^2)\widetilde{q}_{1\rho}\widetilde{q}_{1\rho}, \qquad \eta = \frac{q^2}{2qq_1} > 1\,,
\end{aligned}$$

where we have introduced two dimentionless functions $F_1(\eta,q^2)$ and $F_2(\eta,q^2)$ in a way similar to the DIS case.

By introducing the dimentionless variable $\lambda = 2qq_1/(2zp_1q_1)$, we can write the corresponding cross section for radiative events in e^+e^- annihilation in the same form as in the case of deep inelastic scattering with a tagged photon [4]:

$$\begin{aligned}
&\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{\alpha^2(q^2)}{2\pi}\frac{\alpha}{2\pi}P_1(z,L_0)\Sigma(\eta,\lambda,q^2)\frac{1}{(q^2)^2}\frac{\mathrm{d}^3 q_1}{\varepsilon_1}\,, && (2.9)\\
&\Sigma(\eta,\lambda,q^2) = F_1(\eta,q^2) + \frac{2F_2(\eta,q^2)}{\eta^2\lambda^2}\left(\lambda - 1 - \frac{M^2}{q^2}\eta^2\lambda^2\right).
\end{aligned}$$

3. Radiative corrections

For the radiative corrections (RC) to the cross section (5) we will restrict ourselves only to terms containing second and first powers of large logarithms L, and omit terms which don't contain them i.e. we will keep leading and next–to–leading logarithmic contributions. We will consider in subsection 3.1 the contribution from one–loop virtual photon as well as from the emission of soft real ones. In subsection 3.2 we will discuss the double hard photon emission process.

3.1 Corrections due to the virtual and real soft photons

The interference of Born and one–loop contributions to the amplitude of the initial state radiation in annihilation of e^+e^- into hadrons can be obtained from the analogous quantity of hard photon emission in electron–proton scattering [4]. We do that by using the crossing transformation. For the contribution coming from the emission of real soft photons a straightforward calculation gives:

$$\begin{aligned}
&\frac{\mathrm{d}\sigma^S}{\mathrm{d}\sigma_0} = \frac{\alpha}{\pi}\left[2(L_s-1)\ln\frac{m\Delta\varepsilon}{\lambda\varepsilon} + \frac{1}{2}L_s^2 - \frac{\pi^2}{3}\right], && (3.1)\\
&L_s = \ln\frac{s}{m^2} = L_0 + L_\theta, \qquad L_\theta = \ln\frac{4}{\theta^2}\,.
\end{aligned}$$

where λ is the *photon mass*, $\Delta\varepsilon$ is the energy carried by the soft photon. The sum of the two contributions is free from infrared singularities. It reads

$$\mathrm{d}\sigma^{V+S} = \frac{8\pi^2\alpha^2}{s|1-\Pi(q^2)|^2}\frac{\alpha}{2\pi}\left[\rho B_{\rho\sigma}(q) + A_{\rho\sigma}(q)\right]\frac{H_{\rho\sigma}(q)\mathrm{d}\Gamma(q)}{(q^2)^2}\frac{\alpha}{4\pi^2}\frac{\mathrm{d}^3k}{\omega}\,, \tag{3.2}$$

where

$$\rho = 4(L_s-1)\ln\Delta + 3L_q - \frac{\pi^2}{3} - \frac{9}{2}\,, \qquad L_q = L_s + \ln z, \qquad \Delta = \frac{\Delta\varepsilon}{\varepsilon} \ll 1\,,$$

where k and ω are the 3-momentum and the energy of the hard photon respectively. The tensors $A_{\rho\sigma}$ and $B_{\rho\sigma}$ have a rather involved form. The first can be obtained from the corresponding expressions of ref. [8]. The tensor $B_{\rho\sigma}$ coincides with the one of the Born approximation. In the kinematical region where the hard photon is emitted close to the initial electron direction of motion one has

$$B_{\rho\sigma} = \frac{2}{z}\Big(\frac{1+z^2}{y_1(1-z)} - \frac{2m^2z}{y_1^2}\Big)L_{\rho\sigma}(q)\,, \qquad A_{\rho\sigma} = \frac{2}{q^2}A_g L_{\rho\sigma}(q)\,, \qquad q = zp_1 + p_2, \tag{3.3}$$

where tensor $L_{\rho\sigma}$ is given in eq. (4) , $y_1 = 2kp_1$, and quantity A_g reads

$$\begin{aligned} A_g = {}& \frac{4zsm^2}{y_1^2}L_s\ln z + \frac{s}{y_1}\left[\frac{1+z^2}{1-z}(-2L_s\ln z - \ln^2 z + 2\mathrm{Li}_2(1-z)\right. \\ & \left. + 2\ln\frac{y_1}{m^2}\ln z) + \frac{1+2z-z^2}{2(1-z)}\right]\,, \end{aligned} \tag{3.4}$$

where

$$\mathrm{Li}_2(x) = -\int_0^1 \mathrm{d}t\frac{\ln(1-tx)}{t}\,.$$

Further integration over the hard photon phase space can be performed within the logarithmic accuracy by using the integrals

$$\int\frac{\mathrm{d}^3k}{2\pi k_0}\left[\frac{1}{y_1}, \frac{m^2}{y_1^2}, \frac{\ln(y_1/m^2)}{y_1}\right] = \left[\frac{1}{2}L_0, \frac{1}{2(1-z)}, \frac{1}{4}L_0^2 + \frac{1}{2}L_0\ln(1-z)\right]\mathrm{d}z\,. \tag{3.5}$$

The final expression for the Born cross section corrected for the emission of soft and virtual photons has the form

$$\frac{\mathrm{d}\sigma^{B+V+S}}{\mathrm{d}z} = \sigma_0(z,1)\left[\frac{\alpha}{2\pi}P_1(z,L_0) + \left(\frac{\alpha}{2\pi}\right)^2(\rho P_1(z,L_0) + N)\right]\,, \tag{3.6}$$

where

$$\begin{aligned} N = {}& -\frac{1+z^2}{1-z}\left[(L_0+\ln z)\ln z - \frac{\pi^2}{3} + 2\mathrm{Li}_2(z)\right]L_0 - 2P_1(z,L_0)\ln\frac{\theta_0^2}{4} + \\ & + \frac{1+2z-z^2}{2(1-z)}L_0 + \frac{4z}{1-z}L_0\ln z. \end{aligned}$$

3.2 Two hard photons are tagged by the detector

If an additional hard photon emitted by the initial–state electron hits the PD, we cannot use the quasireal electron method and have to calculate the corresponding contribution starting from Feynman diagrams.

We can use double hard photon spectra as given in ref. [9] for annihilation diagrams only and write the cross section under consideration as follows

$$\frac{\mathrm{d}\sigma_{c1}^{H}}{\mathrm{d}z} = \sigma_0(z,1)\Big(\frac{\alpha}{2\pi}\Big)^2 L_0 \int\limits_{\Delta}^{1-z-\Delta}\frac{\mathrm{d}x}{\xi}\Big[\frac{\gamma\tau}{2}L_0 + (z^2+(1-x)^4)\ln\frac{(1-x)^2(1-z-x)}{zx} + zx(1-z-x) - x^2(1-x-z)^2 - 2\tau(1-x)\Big], \tag{3.7}$$

where

$$\xi = x(1-x)^2(1-z-x), \qquad \gamma = 1+(1-x)^2, \qquad \tau = z^2+(1-x)^2 .$$

Here the variable x under the integral sign is the energy fraction of one hard photon. The quantity $1-z-x$ is the energy fraction of the second hard photon provided that their total energy fraction equals to $1-z$. We write the index $c1$ in the left hand side of eq. (3.7) to emphasize that this contribution arises from the collinear kinematics, when the additional hard photon is emitted along the initial electron with 4–momentum p_1.

The integration in the right hand side of eq. (3.7) leads to the result

$$\frac{\mathrm{d}\sigma_{c1}^{H}}{\mathrm{d}z} = \sigma_0(z,1)\Big(\frac{\alpha}{2\pi}\Big)^2\frac{L_0}{2}\bigg\{\Big[P_{\Theta}^{(2)}(z) + 2\frac{1+z^2}{1-z}\Big(\ln z - \frac{3}{2} - 2\ln\Delta\Big)\Big]L_0 + 6(1-z) + \frac{3+z^2}{1-z}\ln^2 z - \frac{4(1+z)^2}{1-z}\ln\frac{1-z}{\Delta}\bigg\}, \tag{3.8}$$

where the quantity $P_{\Theta}^{(2)}(z)$ represents the so called Θ–term of the second–order electron structure function:

$$P_{\Theta}^{(2)}(z) = 2\frac{1+z^2}{1-z}\Big(\ln\frac{(1-z)^2}{z} + \frac{3}{2}\Big) + (1+z)\ln z - 2(1-z) . \tag{3.9}$$

3.3 Additional hard photon is emitted outside PD

If an additional hard photon, emitted from the initial state, does not hit the PD situated in the direction of motion of the initial electron we distinguish the case when it is emitted in the direction close, within a small cone with angle $\theta' \ll 1$, to the direction of the initial positron.

In this case we obtain:

$$\frac{\mathrm{d}\sigma_{c2}^H}{\mathrm{d}z} = \frac{\alpha}{2\pi}P_1(z,L_0)\int_{\Delta}^{1-\delta/z}\frac{\alpha}{2\pi}P_1(1-x,L')\sigma_0(z,1-x)\mathrm{d}x\,, \tag{3.10}$$

where $L'=L_s+\ln(\theta'^2/4)$, $\delta=M^2/s$, and M^2 is the minimal hadron mass squared. We suppose that $z\sim 1$.

We have introduced the additional auxiliary parameter $\theta'\ll 1$ which, together with θ_0, separates collinear and semi–collinear kinematics of the second hard photon. Contrary to θ_0, which is supposed to determine the PD acceptance, θ' will disappear in the sum of the collinear and semi–collinear contributions of the second photon. This last kinematical region gives

$$\frac{\mathrm{d}\sigma_{sc}^H}{\mathrm{d}z} = \left(\frac{\alpha}{2\pi}\right)^2 P_1(z,L_0)\int\frac{\mathrm{d}^3k_1}{2\pi\omega_1^3}\frac{16\pi^2\alpha^2}{(1-c^2)z^2}T(c,z,x)\,,$$

where

$$T(c,z,x) = \int\frac{H_{\rho\sigma}(q_2)\mathrm{d}\Gamma(q_2)}{s(q_2^2)^2|1-\Pi(q_2^2)|^2}\Big[\frac{s}{2}((z-x_2)^2+z^2(1-x_1)^2)g_{\rho\sigma} + \\ + 2(z(1-x_1)-x_2)(z^2p_{1\rho}p_{1\sigma}+p_{2\rho}p_{2\sigma})\Big]\,,$$

and

$$x_1 = \frac{x}{2}(1-c)\,, \qquad x_2 = \frac{x}{2}(1+c)\,,$$
$$q_2 = zp_1+p_2-k_1\,, \qquad c=\cos\widehat{\boldsymbol{k}_1\boldsymbol{p}_1}\,.$$

The phase volume of the second photon is parametrized as:

$$\int\frac{\mathrm{d}^3k_1}{2\pi\omega^3} = \int\limits_{\Delta}^{\hat{x}}\frac{\mathrm{d}x}{x}\int\limits_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\int\limits_{-1+\theta'^2/2}^{1-\theta_0^2/2}\mathrm{d}c\,, \qquad \hat{x}=\frac{2(z-\delta)}{1+z+c(1-z)}\,. \tag{3.11}$$

Explicitly extracting the angular singularities we represent this expression as

$$\frac{\mathrm{d}\sigma_{sc}^H}{\mathrm{d}z} = \left(\frac{\alpha}{2\pi}\right)^2 P_1(z,L_0)\Bigg[\Sigma_{sc}(z)+\ln\frac{4}{\theta_0^2}\int\limits_{\Delta}^{z-\delta}\frac{\mathrm{d}x}{x}\frac{z^2+(z-x)^2}{z^2}\sigma_0(z-x,1)+ \\ +\ln\frac{4}{\theta'^2}\int\limits_{\Delta}^{1-\delta/z}\frac{\mathrm{d}x}{x}(1+(1-x)^2)\sigma_0(z,1-x)\Bigg]\,, \tag{3.12}$$

where

$$\Sigma_{sc} = \frac{8\pi^2\alpha^2}{z^2}\int\limits_{-1}^{1}\mathrm{d}c\int\limits_{\Delta}^{\hat{x}}\frac{\mathrm{d}x}{x}\left[\frac{T(c,z,x)-T(1,z,x)}{1-c}+\frac{T(c,z,x)-T(-1,z,x)}{1+c}\right]\,.$$

4. Complete QED correction and leading logarithmic approximation

The final result in the order $\mathcal{O}(\alpha)$ for radiative corrections to radiative events can be written as follows:

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}z} &= \frac{\alpha}{2\pi}P_1(z,L_0)\sigma_0(z,1)(1+r) \\
&= \frac{\alpha}{2\pi}P_1(z,L_0)\sigma_0(z,1)+\left(\frac{\alpha}{2\pi}\right)^2\Bigg\{L_0\left(\frac{1}{2}L_0P^{(2)}(z)+G\right)\sigma_0(z,1)+P_1(z,L_0)\times \\
&\times\left[\int\limits_0^{1-\delta/z}C_1(x)\sigma_0(z,1-x)\mathrm{d}x+L_\theta\int\limits_0^{z-\delta}C_2(z,x)\sigma_0(z-x,1)\mathrm{d}x+\Sigma_{sc}\right]\Bigg\}\,,
\end{aligned}
\tag{4.1}
$$

where the last term is defined by eq. (23) and

$$
\begin{aligned}
C_1(x) &= P_1(1-x,L_s)\Theta(x-\Delta)+(L_s-1)(2\ln\Delta+\frac{3}{2})\delta(x)\,, \\
C_2(z,x) &= \frac{z^2+(z-x)^2}{z^2x}\Theta(x-\Delta)+(2\ln\Delta+\frac{3}{2}-2\ln z)\delta(x)\,, \\
G(z) &= \frac{1+z^2}{1-z}(3\ln z-2\mathrm{Li}_2(z))+\frac{1}{2}(1+z)\ln^2 z-\frac{2(1+z)^2}{1-z}\ln(1-z)+ \\
&+\frac{1-16z-z^2}{2(1-z)}+\frac{4z\ln z}{1-z}\,.
\end{aligned}
\tag{4.2}
$$

In order to include the higher order leading corrections to the tagged photon differential cross–section and show the agreement of our calculation with the well known Drell–Yan representation for the total hadronic cross–section at electron–positron annihilation [10]

$$
\sigma(s)=\int\limits_\delta^1\mathrm{d}x_1\int\limits_{\delta/x_1}^1\mathrm{d}x_2\;D(x_1,\alpha_{\mathrm{eff}})D(x_2,\alpha_{\mathrm{eff}})\sigma(x_1x_2s)\,, \tag{4.3}
$$

where the electron structure functions include both nonsinglet and singlet parts

$$
D(x_1,\alpha_{\mathrm{eff}})=D^{NS}(x,\alpha_{\mathrm{eff}})+D^S(x_1,\alpha_{\mathrm{eff}})\,, \tag{4.4}
$$

it is convenient to introduce the quantity

$$
\Sigma=D(z,\bar{\alpha}_{\mathrm{eff}})\int\limits_{\delta/z}^1\mathrm{d}x_1\int\limits_{\delta/zx_1}^1\mathrm{d}x_2\;D(x_1,\tilde{\alpha}_{\mathrm{eff}})D(x_2,\hat{\alpha}_{\mathrm{eff}})\sigma_0(zx_1,x_2)\,. \tag{4.5}
$$

Note that the shifted cross–section in eq. (4.3) has just the same meaning as in eq. (2.3): $\sigma(x_1x_2s)=\sigma_0(x_1,x_2)$.

The structure functions [11, 12], which enter into the right side of eq. (4.4), are

$$D^{NS}(x,\alpha_{\rm eff}) = \delta(1-x) + \sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\alpha_{\rm eff}}{2\pi}\right)^n P_1^{\otimes n}(x) \tag{4.6}$$

and

$$D^{S}(x,\alpha_{\rm eff}) = \frac{1}{2!}\left(\frac{\alpha_{\rm eff}}{2\pi}\right)^2 R(x) + \frac{1}{3!}\left(\frac{\alpha_{\rm eff}}{2\pi}\right)^3\left[2P_1\otimes R(x) - \frac{2}{3}R(x)\right], \tag{4.7}$$

where

$$\begin{aligned} P_1(x) &= \lim_{\Delta\to 0}\left\{\frac{1+x^2}{1-x}\Theta(1-\Delta-x) + \left(\frac{3}{2}+2\ln\Delta\right)\delta(1-x)\right\}, \\ R(x) &= 2(1+x)\ln x + \frac{1-x}{3x}(4+7x+4x^2), \\ P_1^{\otimes n} &= \underbrace{P_1(x)\otimes\cdots\otimes P_1(x)}_{n}, \qquad P_1(x)\otimes P_1(x) = \int\limits_x^1 P_1(t)P_1\left(\frac{x}{t}\right)\frac{{\rm d}t}{t}, \end{aligned}$$

and the effective electromagnetic couplings in the right side of eq. (4.5) are

$$\bar{\alpha}_{\rm eff} = -3\pi\ln\left(1-\frac{\alpha}{3\pi}L_0\right), \quad \tilde{\alpha}_{\rm eff} = -3\pi\ln\left(\frac{1-\frac{\alpha}{3\pi}L_s}{1-\frac{\alpha}{3\pi}L_0}\right), \quad \hat{\alpha}_{\rm eff} = -3\pi\ln\left(1-\frac{\alpha}{3\pi}L_s\right). \tag{4.8}$$

At fixed values of z $(z<1)$ the quantity Σ defines the leading logarithmic contributions into differential cross–section for the events with tagged particles. That corresponds to only Θ–terms in the expansion of the structure function $D(z,\bar{\alpha}_{\rm eff})$ before the integral sign in eq. (4.5). If we consider photonic corrections (as in previous Sections) it needs to restrict ourselves with the nonsinglet part of the electron structure functions and with the first order terms in the expansion of all effective couplings, namely:

$$\bar{\alpha}_{\rm eff}\to\alpha L_0\,, \qquad \tilde{\alpha}_{\rm eff}\to\alpha L_\theta\,, \qquad \hat{\alpha}_{\rm eff}\to\alpha L_s\,. \tag{4.9}$$

It is easy to see that in this case the leading contribution into differential cross–section (4.1) can be obtained as an expansion of the quantity $\Sigma(z<1)$ by the powers of α, keeping the terms of the order α^2 in the production of D–functions.

If we want to include the contribution due to e^+e^-–pair (real and virtual) production it is required [14] to use both nonsinglet and singlet structure functions and effective couplings defined by eq. (4.8). Note that the insertion into consideration of higher order corrections rises additional questions about concrete experimental conditions concerning registration of events with e^+e^-–pairs.

The total hadronic cross–section in e^+e^- annihilation can be obtained by integration of quantity Σ over z

$$\sigma(s) = \int\limits_{\delta}^{1}{\rm d}z\;D(z,\bar{\alpha}_{\rm eff})\int\limits_{\delta/z}^{1}{\rm d}x_1\int\limits_{\delta/zx_1}^{1}{\rm d}x_2\;D(x_1,\tilde{\alpha}_{\rm eff})D(x_2,\hat{\alpha}_{\rm eff})\sigma(zx_1x_2s)\,. \tag{4.10}$$

We can integrate the expression in the right side of eq. (4.10) over the variable z provided the quantity $zx_1 = y$ is fixed

$$\int\limits_{\delta}^{1} \mathrm{d}z\, D(z,\bar{\alpha}_{\text{eff}}) \int\limits_{\delta/z}^{1} \mathrm{d}x_1\, D(x_1,\tilde{\alpha}_{\text{eff}}) = \int\limits_{\delta}^{1} \mathrm{d}z \int\limits_{y}^{1} \mathrm{d}y\, D(z,\bar{\alpha}_{\text{eff}}) D\Big(\frac{y}{z},\tilde{\alpha}_{\text{eff}}\Big) \qquad (4.11)$$

$$= \int\limits_{\delta}^{1} \mathrm{d}y\, D(y,\bar{\alpha}_{\text{eff}}+\tilde{\alpha}_{\text{eff}})\,,$$

and $\bar{\alpha}_{\text{eff}} + \tilde{\alpha}_{\text{eff}} = \hat{\alpha}_{\text{eff}}$.

Using this result and definition of $\hat{\alpha}_{\text{eff}}$ we indicate the equivalence of the Drell–Yan form of the total cross–section as given by eq. (4.3) and the representation of the cross–section by eq. (4.10).

Let us show now that D–functions in expression for the quantity Σ have effective couplings as given by eq. (4.8). By definition the nonsinglet electron structure function satisfies the equation [13]

$$D(x,s,s_0) = \delta(1-x) + \frac{1}{2\pi} \int\limits_{s_0}^{s} \frac{ds_1}{s_1} \alpha(s_1) \int\limits_{x}^{1} \frac{dz}{z} D(z) D\Big(\frac{x}{z},\frac{s_1}{s_0}\Big)\,, \qquad (4.12)$$

where $\alpha(s_1)$ is the electromagnetic running coupling

$$\alpha(s_1) = \alpha\Big(1 - \frac{\alpha}{3\pi}\ln\frac{s_1}{m^2}\Big)^{-1}\,,$$

and $s_0(s)$ is the minimal (maximal) virtuality of the particle, which radiate photons and e^+e^-–pairs.

The structure function $D(z,\bar{\alpha}_{\text{eff}})$ describes the photon emission and pair production inside narrow cone along the electron beam direction. In this kinematics $s_0 = m^2$, $s = \varepsilon^2\theta_0^2$. The corresponding iterative solution of the eq. (4.12) has the form (4.6) with $\alpha_{\text{eff}} = \bar{\alpha}_{\text{eff}}$. The structure function $D(x_1,\tilde{\alpha}_{\text{eff}})$ describes the events, when emitted (by the electron) particles escape this narrow cone. In this case $s_0 = \varepsilon^2\theta_0^2$, $s = 4\varepsilon^2$. The corresponding solution of eq. (4.12) gives the structure function with $\alpha_{\text{eff}} = \tilde{\alpha}_{\text{eff}}$. At last, the structure function $D(x_2,\hat{\alpha}_{\text{eff}})$ is responsible for the radiation off the positron into the whole phase space. In this case $s_0 = m^2$, $s = 4\varepsilon^2$. Therefore we obtain D–function with $\alpha_{\text{eff}} = \hat{\alpha}_{\text{eff}}$. The analogous consideration can be performed for the singlet part of structure functions.

When writing the representation (4.10) for the total cross–section we, in fact, divide the phase space of the particles emitted by the electron on the regions inside and outside the narrow cone along electron beam direction. Therefore we can use this representation to investigate the events with tagged particles in both this regions. As we saw before the differential cross–section for events with tagged particles inside the narrow cone is defined by the quantity $\Sigma(z<1)$. In order to obtain the corresponding differential cross–section for events with tagged particles outside this narrow cone we have to change the places of $\bar{\alpha}_{\text{eff}}$ and $\tilde{\alpha}_{\text{eff}}$ in expression for $\Sigma(z,1)$. This follows from the symmetry of representation (4.10) relative such change.

5. Conclusion

In sum the formulae (4.11,4.5) are the main results of our paper. In fig. 1 we show the cross section $\mathrm{d}\sigma/\mathrm{d}z$ as a function of z.

The beam energy is chosen to be $E_{\text{beam}} = 0.5$ GeV. The region of z values is limited by the pion production threshold at the left, and by the threshold of photon detection (we choose 50 MeV) at the right. The peak in middle corresponds to the large contribution of the ρ-meson. Values of R, used for numerical estimations, were taken from [15]. The values of corrections r (see eq. (4.11)) in percent are shown in fig. 2.

So, we calculated the cross–section of e^+e^- annihilation with detection of a hard photon at small angles in respect to the electron beam. The general structure of a measured cross–section, from which one should extract the annihilation crossection σ_0, looks as follows:

$$\sigma = \sigma_0 \left[a_1 \frac{\alpha}{\pi} L + b_1 \frac{\alpha}{\pi} + a_2 \left(\frac{\alpha}{\pi}\right)^2 L^2 + b_2 \left(\frac{\alpha}{\pi}\right)^2 L + c_2 \left(\frac{\alpha}{\pi}\right)^2 \right] + \mathcal{O}(\alpha^3)\,, \tag{5.1}$$

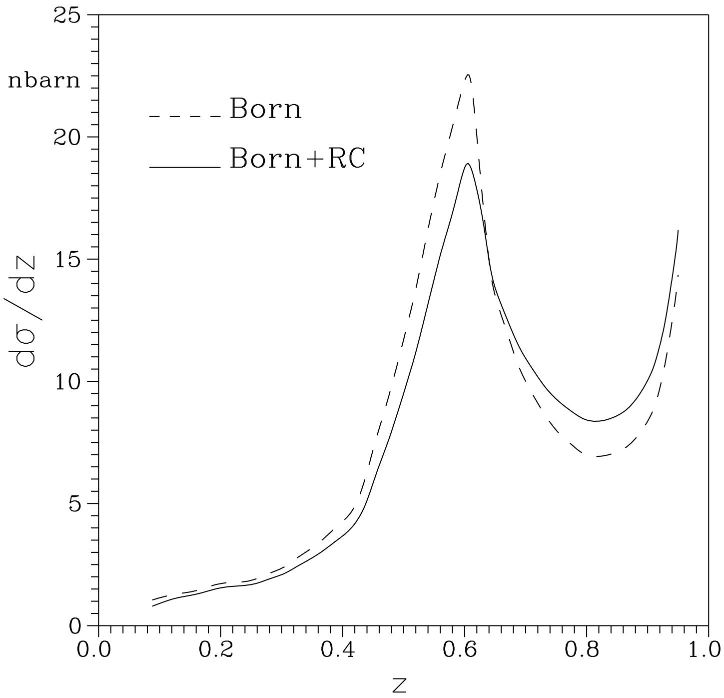

Figure 1: The cross section of $e^+e^- \to$ hadrons with tagged photon.

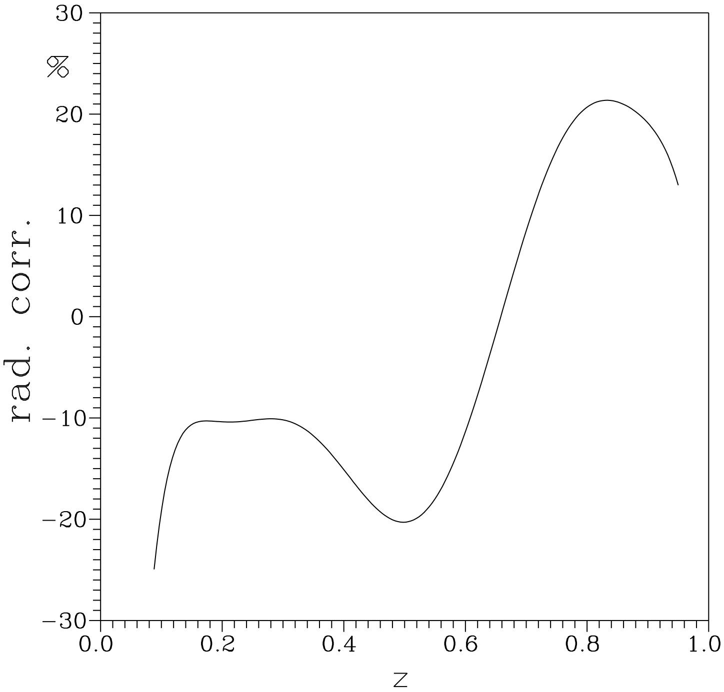

Figure 2: The radiative corrections to $e^+e^- \to$ hadrons with tagged photon.

where L denotes some large logarithm. We calculated the terms a_1, b_1, a_2, b_2 and some contributions to c_2. The generalized formula (4.5) allows to involve the leading terms of the order $\mathcal{O}(\alpha^3 L^3)$. In this way our formulae provide high theoretical precision.

Similar formulae can be obtained for an experimental set–up by tagging a definite hadron. By using e^+e^-–machines such as BEPS, DAΦNE [16], VEPP, CLEO, SLAC–B/factory and others with luminosities of order $10^{33} cm^2 s^{-1}$, one may be in principle able to scan, by measuring the initial state radiation spectrum, the whole energy region of hadron production with an effective luminosity of the order of $10^{31} cm^2 s^{-1}$. We hope further study will follow on these issues both from the experiments and theory.

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